A Way of Reasoning for Selecting One and Rejecting the Other of the Two Roots Which Arise from the Square-law Relationship between the Current and the Voltage of a MOSFET in the Large-signal Analysis of a MOSFET Differential Amplifier Without Taking Channel-length Modulation into Account

by

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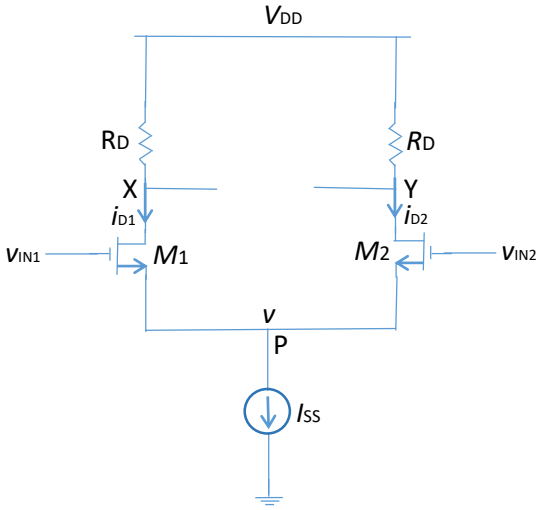

Fig. 1: Circuit Diagram of the MOSFET Differential Amplifier

We assume that in the MOSFET differential amplifier circuit of Fig. 1, MOSFETs M_1 and M_2 are matched.

The equations relating the currents and voltages of the above circuit (shown in Fig. 1) are as follows:

$$i_{D1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{GS1} - V_{Th})^2 \ldots (1)$$

$$or, i_{D1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{IN1} - v - V_{Th})^2 \ldots (2)\ [since\ v_{GS1} = v_{IN1} - v]$$

$$i_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{GS2} - V_{Th})^2 \ldots (3)$$

$$or, i_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{IN2} - v - V_{Th})^2 \ldots (4)\ [since\ v_{GS2} = v_{IN2} - v]$$

$$i_{D1} + i_{D2} = I_{SS} \ldots (5)$$

$$v_X = V_{DD} - i_{D1}R_D \ldots (6)$$

$$v_Y = V_{DD} - i_{D2}R_D \ldots (7)$$

Now, to solve the above equations:

Equations (2), (4) and (5) =>

$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{IN1}-v-V_{Th})^2+\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{IN2}-v-V_{Th})^2=I_{SS}$$

$$or,(v_{IN1}-v-V_{Th})^2+(v_{IN2}-v-V_{Th})^2=\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$$

$$or,[(v_{IN1}-V_{Th})-v]^2+[(v_{IN2}-V_{Th})-v]^2=\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$$

$$or,(v_{IN1}-V_{Th})^2+v^2-2(v_{IN1}-V_{Th})v$$
$$+(v_{IN2}-V_{Th})^2+v^2-2(v_{IN2}-V_{Th})v=\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$$

$$or,2v^2-2(v_{IN1}+v_{IN2}-2V_{Th})v$$
$$+\left[(v_{IN1}-V_{Th})^2+(v_{IN2}-V_{Th})^2-\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}\right]=0\ldots(8)$$

$$\therefore v=\frac{2(v_{IN1}+v_{IN2}-2V_{Th})\pm\sqrt{4(v_{IN1}+v_{IN2}-2V_{Th})^2-8\left[(v_{IN1}-V_{Th})^2+(v_{IN2}-V_{Th})^2-\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}\right]}}{4}$$

$$or,v=\frac{2(v_{IN1}+v_{IN2}-2V_{Th})\pm 2\sqrt{(v_{IN1}+v_{IN2}-2V_{Th})^2-2\left[(v_{IN1}-V_{Th})^2+(v_{IN2}-V_{Th})^2-\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}\right]}}{4}$$

$$or,v=\frac{(v_{IN1}+v_{IN2}-2V_{Th})\pm\sqrt{(v_{IN1}+v_{IN2}-2V_{Th})^2-2\left[(v_{IN1}-V_{Th})^2+(v_{IN2}-V_{Th})^2-\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}\right]}}{2}\ldots(9)$$

$$(v_{IN1}+v_{IN2}-2V_{Th})^2-2\left[(v_{IN1}-V_{Th})^2+(v_{IN2}-V_{Th})^2-\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}\right]$$

$$=v_{IN1}{}^2+v_{IN2}{}^2+4V_{Th}{}^2+2v_{IN1}v_{IN2}-4v_{IN1}V_{Th}-4v_{IN2}V_{Th}-2\left[v_{IN1}{}^2+V_{Th}{}^2-\right.$$

$$\left.2v_{IN1}V_{Th}+v_{IN2}{}^2+V_{Th}{}^2-2v_{IN2}V_{Th}-\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}\right]$$

$$=v_{IN1}{}^2+v_{IN2}{}^2+4V_{Th}{}^2+2v_{IN1}v_{IN2}-4v_{IN1}V_{Th}-4v_{IN2}V_{Th}-2\left[v_{IN1}{}^2+v_{IN2}{}^2+\right.$$

$$\left.2V_{Th}{}^2-2v_{IN1}V_{Th}-2v_{IN2}V_{Th}-\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}\right]$$

$= {v_{IN1}}^2 + {v_{IN2}}^2 + 4{V_{Th}}^2 + 2v_{IN1}v_{IN2} - 4v_{IN1}V_{Th} - 4v_{IN2}V_{Th} - 2{v_{IN1}}^2 - 2{v_{IN2}}^2 - 4{V_{Th}}^2 + 4v_{IN1}V_{Th} + 4v_{IN2}V_{Th} + \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$

$= \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - {v_{IN1}}^2 - {v_{IN2}}^2 + 2v_{IN1}v_{IN2}$

$= \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - ({v_{IN1}}^2 + {v_{IN2}}^2 - 2v_{IN1}v_{IN2})$

$= \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2$

$\therefore (v_{IN1} + v_{IN2} - 2V_{Th})^2 - 2\left[(v_{IN1} - V_{Th})^2 + (v_{IN2} - V_{Th})^2 - \frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}\right] = \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2 \ldots (10)$

Equations (9) and (10) =>

$$v = \frac{(v_{IN1}+v_{IN2}-2V_{Th}) \pm \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2}}{2} \ldots (11)$$

$\therefore$ The two possible roots of v are

$$v_1 = \frac{(v_{IN1}+v_{IN2}-2V_{Th}) + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2}}{2} \ldots (12a)$$

and

$$v_2 = \frac{(v_{IN1}+v_{IN2}-2V_{Th}) - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2}}{2} \ldots (12b)$$

Now, to test the admissibility of the root v_1 given by equation (12*a*):

$$v = v_1 = \frac{(v_{IN1}+v_{IN2}-2V_{Th}) + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2}}{2} \ldots (12a)$$

$v_{GS1} = v_{IN1} - v$

$= v_{IN1} - v_1$

$$= v_{IN1} - \frac{(v_{IN1}+v_{IN2}-2V_{Th}) + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2}}{2}$$

$$= \frac{2v_{IN1} - (v_{IN1} + v_{IN2} - 2V_{Th}) - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}}{2}$$

$$= \frac{2v_{IN1} - v_{IN1} - v_{IN2} + 2V_{Th} - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}}{2}$$

$$or, v_{GS1} = \frac{(v_{IN1} - v_{IN2}) + 2V_{Th} - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}}{2} \ldots (13)$$

$$\therefore\ v_{GS1} - V_{Th} = \frac{(v_{IN1} - v_{IN2}) + 2V_{Th} - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}}{2} - V_{Th}$$

$$= \frac{(v_{IN1} - v_{IN2}) + 2V_{Th} - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2} - 2V_{Th}}{2}$$

$$or, v_{GS1} - V_{Th} = \frac{(v_{IN1} - v_{IN2}) - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}}{2} \ldots (14)$$

For the MOSFET M_1 to operate, we require that

$$v_{GS1} - V_{Th} \geq 0 \ldots (15)$$

$$or, (v_{IN1} - v_{IN2}) - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}\ \geq 0$$

$$or, (v_{IN1} - v_{IN2}) \geq \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2} \ldots (16)$$

$$or, (v_{IN1} - v_{IN2})^2 \geq \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2$$

$$or, 2(v_{IN1} - v_{IN2})^2 \geq \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$$

$$or, (v_{IN1} - v_{IN2})^2 \geq \frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}} \ldots (17)$$

$$\therefore\ v_{IN1} - v_{IN2} \geq \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (18a)$$

OR

$$v_{IN1} - v_{IN2} \leq -\sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (18b)$$

Inequality (18*b*) is inadmissible because of inequality (16).

$$\therefore\ v_{IN1} - v_{IN2} \geq \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (19)$$

Equation (14) =>

For $(v_{GS1} - V_{Th})$ to be real, we require that

the expression under the radical sign of equation (14) ≥ 0.

$$\therefore\ \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2 \geq 0$$

$$or, (v_{IN1} - v_{IN2})^2 \leq \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} \ldots (20)$$

$$\therefore\ -\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \leq v_{IN1} - v_{IN2} \leq \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (21)$$

Inequalities (19) and (21) must be satisfied simultaneously.

$$\therefore\ \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \leq v_{IN1} - v_{IN2} \leq \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (22)$$

$$v_{GS2} = v_{IN2} - v$$

$$= v_{IN2} - v_1$$

$$= v_{IN2} - \frac{(v_{IN1} + v_{IN2} - 2V_{Th}) + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}}{2}$$

$$= \frac{2v_{IN2} - (v_{IN1} + v_{IN2} - 2V_{Th}) - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}}{2}$$

$$= \frac{2v_{IN2} - v_{IN1} - v_{IN2} + 2V_{Th} - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}}{2}$$

$$or, v_{GS2} = \frac{(v_{IN2} - v_{IN1}) + 2V_{Th} - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}}{2} \ldots (23)$$

$$or, v_{GS2} = \frac{(v_{IN2} - v_{IN1}) + 2V_{Th} - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN2} - v_{IN1})^2}}{2} \ldots (24)$$

$$\therefore\ v_{GS2} - V_{Th} = \frac{(v_{IN2}-v_{IN1})+2V_{Th}-\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(v_{IN2}-v_{IN1})^2}}{2} - V_{Th}$$

$$= \frac{(v_{IN2}-v_{IN1})+2V_{Th}-\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(v_{IN2}-v_{IN1})^2}-2V_{Th}}{2}$$

$$or, v_{GS2} - V_{Th} = \frac{(v_{IN2}-v_{IN1})-\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(v_{IN2}-v_{IN1})^2}}{2} \ldots (25)$$

$$or, v_{GS2} - V_{Th} = \frac{-(v_{IN1}-v_{IN2})-\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(v_{IN1}-v_{IN2})^2}}{2} \ldots (26)$$

Equation (26) =>

In the domain (22),

$v_{GS2} - V_{Th} < 0 \ldots (27)$

$\therefore$ When $(v_{GS1} - V_{Th}) \geq 0$, then $(v_{GS2} - V_{Th}) < 0 \ldots (28)$

For MOSFET M_2 to operate, we must have

$v_{GS2} - V_{Th} \geq 0 \ldots (29)$

$$or, (v_{IN2} - v_{IN1}) - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN2} - v_{IN1})^2} \geq 0\ [by\ (25)]$$

$$or, v_{IN2} - v_{IN1} \geq \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN2} - v_{IN1})^2} \ldots (30)$$

$$or, (v_{IN2} - v_{IN1})^2 \geq \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN2} - v_{IN1})^2$$

$$or, 2(v_{IN2} - v_{IN1})^2 \geq \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$$

$$or, (v_{IN2} - v_{IN1})^2 \geq \frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}} \ldots (31)$$

$$\therefore\ v_{IN2} - v_{IN1} \geq \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (32a)$$

OR

$$v_{IN2} - v_{IN1} \leq -\sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (32b)$$

Inequality (32b) is inadmissible because of inequality (30).

$\therefore\ v_{IN2} - v_{IN1} \geq \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (33)$

Equation (25) =>

For $(v_{GS2} - V_{Th})$ to be real, we require that

the expression under the radical sign of equation (25) ≥ 0.

$\therefore\ \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN2} - v_{IN1})^2 \geq 0$

$or, (v_{IN2} - v_{IN1})^2 \leq \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} \ldots (34)$

$\therefore\ -\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \leq v_{IN2} - v_{IN1} \leq \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (35)$

Since inequalities (33) and (35) must be satisfied simultaneously,

$\therefore\ \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \leq v_{IN2} - v_{IN1} \leq \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (36)$

Equation (14) =>

$$v_{GS1} - V_{Th} = \frac{-(v_{IN2}-v_{IN1}) - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN2}-v_{IN1})^2}}{2} \ldots (37)$$

Equation (37) =>

In the domain (36),

$v_{GS1} - V_{Th} < 0 \ldots (38)$

$\therefore$ When $(v_{GS2} - V_{Th}) \geq 0$, then $(v_{GS1} - V_{Th}) < 0 \ldots (39)$

Equations (28) and (39) =>

For $v = v_1$ (given by equation (12a)),

when $(v_{GS1} - V_{Th}) \geq 0$, then $(v_{GS2} - V_{Th}) < 0$,

and when $(v_{GS2} - V_{Th}) \geq 0$, then $(v_{GS1} - V_{Th}) < 0$.

Thus, we see that for $v = v_1$ (given by equation (12a)), when MOSFET M_1 is on, MOSFET M_2 is off, and when MOSFET M_2 is on, MOSFET M_1 is off. Hence, the square-law relating the currents and the voltages of MOSFETs M_1 and M_2, which gave us the root v_1, does not hold for the MOSFET which is off. Hence, the root $v = v_1$ given by equation (12a) is inadmissible.

Now, let us see what happens when one of the MOSFETs is on and the other is off.

Say, M_2 is off. Then, the maximum value which v_{GS2} can have is V_{Th}, i.e.,

$v_{GS2} \leq V_{Th} \ldots (40)$

But then, the whole of the current I_{SS} must flow through M_1. We then have

$I_{SS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{GS1} - V_{Th})^2 \ldots (41)$

$or, (v_{GS1} - V_{Th})^2 = \frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$

$or, v_{GS1} - V_{Th} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$

$\therefore\ v_{GS1} = V_{Th} + \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (42)$

$or, v_{IN1} - v = V_{Th} + \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}\ [since\ v_{GS1} = v_{IN1} - v]$

$\therefore v = v_{IN1} - \left(V_{Th} + \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}\right) \ldots (43)$

Equation (40) =>

$v_{IN2} - v \leq V_{Th} \ldots (44)\ [since\ v_{GS2} = v_{IN2} - v]$

$or, v_{IN2} \leq v + V_{Th}$

$or, v_{IN2} \leq v_{IN1} - \left(V_{Th} + \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}\right) + V_{Th}\ [using\ (43)]$

$or, v_{IN2} \leq v_{IN1} - V_{Th} - \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} + V_{Th}$

$or, v_{IN2} \leq v_{IN1} - \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$

$or, v_{IN1} - v_{IN2} \geq \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (45)$

In this case,

$i_{D1} = I_{SS} \ldots (46)$

$i_{D2} = 0 \ldots (47)$

Equation (6) =>

$v_X = V_{DD} - i_{D1}R_D$

$or, v_X = V_{DD} - I_{SS}R_D \ldots (48)\ [using\ (46)]$

Equation (7) =>

$v_Y = V_{DD} - i_{D2}R_D$

$or, v_Y = V_{DD} - 0\ [using\ (47)]$

$or, v_Y = V_{DD} \ldots (49)$

(48) – (49) =>

$v_X - v_Y = -I_{SS}R_D \ldots (50)$

Now, say M_1 is off. Then, the maximum value which v_{GS1} can have is V_{Th}, i.e.,

$v_{GS1} \leq V_{Th} \ldots (51)$

But then, the whole of the current I_{SS} must flow through M_2. We then have

$I_{SS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{GS2} - V_{Th})^2 \ldots (52)$

$or, (v_{GS2} - V_{Th})^2 = \frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$

$or, v_{GS2} - V_{Th} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$

$\therefore\ v_{GS2} = V_{Th} + \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (53)$

$or, v_{IN2} - v = V_{Th} + \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}\ [since\ v_{GS2} = v_{IN2} - v]$

$\therefore v = v_{IN2} - \left(V_{Th} + \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}\right) \ldots (54)$

Equation (51) =>

$v_{IN1} - v \leq V_{Th} \ldots (55)\ [since\ v_{GS1} = v_{IN1} - v]$

$or, v_{IN1} \leq v + V_{Th}$

$$or, v_{IN1} \leq v_{IN2} - \left(V_{Th} + \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}\right) + V_{Th}\ [by\ (54)]$$

$$or, v_{IN1} \leq v_{IN2} - V_{Th} - \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} + V_{Th}$$

$$or, v_{IN1} \leq v_{IN2} - \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$$

$$or, v_{IN2} - v_{IN1} \geq \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (56)$$

$$or, v_{IN1} - v_{IN2} \leq -\sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (57)$$

In this case,

$i_{D1} = 0 \ldots (58)$

$i_{D2} = I_{SS} \ldots (59)$

Equation (6) =>

$v_X = V_{DD} - i_{D1}R_D$

$or, v_X = V_{DD} - 0\ [using\ (58)]$

$or, v_X = V_{DD} \ldots (60)$

Equation (7) =>

$v_Y = V_{DD} - i_{D2}R_D$

$or, v_Y = V_{DD} - I_{SS}R_D \ldots (61)\ [using\ (59)]$

(60) – (61) =>

$v_X - v_Y = I_{SS}R_D \ldots (62)$

Thus, we have the following table (Table-1) which gives the values of the currents and voltages in the circuit in the regions $(v_{IN1} - v_{IN2}) \geq \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$ and $(v_{IN1} - v_{IN2}) \leq -\sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$.

Table-1: Values of the Currents and Voltages in the Circuit in the Regions $(v_{IN1} - v_{IN2}) \geq \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$ and $(v_{IN1} - v_{IN2}) \leq -\sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$

Variable	Region	
	$(v_{IN1} - v_{IN2}) \geq \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$ **(M_2 off, M_1 on)**	$(v_{IN1} - v_{IN2}) \leq -\sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$ **(M_1 off, M_2 on)**
v	$v_{IN1} - \left(V_{Th} + \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}\right)$	$v_{IN2} - \left(V_{Th} + \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}\right)$
i_{D1}	I_{SS}	0
i_{D2}	0	I_{SS}
v_X	$V_{DD} - I_{SS}R_D$	V_{DD}
v_Y	V_{DD}	$V_{DD} - I_{SS}R_D$
$v_X - v_Y$	$-I_{SS}R_D$	$I_{SS}R_D$

Now, to test the admissibility of the root v_2 given by equation (12*b*):

$$v = v_2 = \frac{(v_{IN1}+v_{IN2}-2V_{Th}) - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2}}{2} \ldots (12b)$$

$$v_{GS1} = v_{IN1} - v$$

$$= v_{IN1} - v_2$$

$$= v_{IN1} - \frac{(v_{IN1}+v_{IN2}-2V_{Th}) - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2}}{2}$$

$$= \frac{2v_{IN1} - (v_{IN1}+v_{IN2}-2V_{Th}) + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2}}{2}$$

$$= \frac{2v_{IN1} - v_{IN1} - v_{IN2} + 2V_{Th} + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2}}{2}$$

$$or, v_{GS1} = \frac{(v_{IN1}-v_{IN2}) + 2V_{Th} + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2}}{2} \ldots (63)$$

$$\therefore\ v_{GS1} - V_{Th} = \frac{(v_{IN1}-v_{IN2})+2V_{Th}+\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(v_{IN1}-v_{IN2})^2}}{2} - V_{Th}$$

$$= \frac{(v_{IN1}-v_{IN2})+2V_{Th}+\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(v_{IN1}-v_{IN2})^2}-2V_{Th}}{2}$$

$$or, v_{GS1} - V_{Th} = \frac{(v_{IN1}-v_{IN2})+\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(v_{IN1}-v_{IN2})^2}}{2} \dots (64)$$

For MOSFET M_1 to operate, we must have

$$v_{GS1} - V_{Th} \geq 0 \dots (65)$$

$$or, (v_{IN1} - v_{IN2}) + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2} \geq 0 \dots (66)$$

Case-I: $v_{IN1} - v_{IN2} \geq 0$

Let, $v_{IN1} - v_{IN2} = +x \dots (67)$ $[where\ x \geq 0]$

Then, equation (66) becomes

$$x + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - x^2} \geq 0 \dots (68)$$

Inequality (68) holds for

$$0 \leq x \leq \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \dots (69)$$

$$or, 0 \leq v_{IN1} - v_{IN2} \leq \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \dots (70)\ [by\ (67)]$$

Case-II: $v_{IN1} - v_{IN2} < 0$

Let, $v_{IN1} - v_{IN2} = -x \dots (71)$ $[where\ x > 0]$

Then, equation (66) becomes

$$-x + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - x^2} \geq 0$$

$$or, \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - x^2} \geq x$$

$$or, x \leq \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - x^2}$$

$$or, x^2 \leq \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - x^2$$

$$or, 2x^2 \leq \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$$

$$or, x^2 \leq \frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$$

$$\therefore 0 < x \leq \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (72)$$

$$or, 0 > -x \geq -\sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$$

$$or, -\sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \leq -x < 0 \ldots (73)$$

$$or, -\sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \leq v_{IN1} - v_{IN2} < 0 \ldots (74)\ [by\ (71)]$$

Inequalities (70) and (74) =>

$$-\sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \leq v_{IN1} - v_{IN2} \leq \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (75)$$

$$v_{GS2} = v_{IN2} - v$$

$$= v_{IN2} - v_2$$

$$= v_{IN2} - \frac{(v_{IN1}+v_{IN2}-2V_{Th}) - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2}}{2}$$

$$= \frac{2v_{IN2} - (v_{IN1}+v_{IN2}-2V_{Th}) + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2}}{2}$$

$$= \frac{2v_{IN2} - v_{IN1} - v_{IN2} + 2V_{Th} + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2}}{2}$$

$$or, v_{GS2} = \frac{(v_{IN2}-v_{IN1}) + 2V_{Th} + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2}}{2} \ldots (76)$$

$$\therefore\ v_{GS2} - V_{Th} = \frac{(v_{IN2}-v_{IN1}) + 2V_{Th} + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2}}{2} - V_{Th}$$

$$= \frac{(v_{IN2}-v_{IN1})+2V_{Th}+\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(v_{IN1}-v_{IN2})^2}-2V_{Th}}{2}$$

$$or, v_{GS2} - V_{Th} = \frac{(v_{IN2}-v_{IN1})+\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(v_{IN1}-v_{IN2})^2}}{2} \ldots (77)$$

$$or, v_{GS2} - V_{Th} = \frac{-(v_{IN1}-v_{IN2})+\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(v_{IN1}-v_{IN2})^2}}{2} \ldots (78)$$

For MOSFET M_2 to operate, we must have

$v_{GS2} - V_{Th} \geq 0 \ldots (79)$

$$or, -(v_{IN1} - v_{IN2}) + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2} \geq 0 \ldots (80)$$

Case-I: $v_{IN1} - v_{IN2} \geq 0$

Let, $v_{IN1} - v_{IN2} = +x \ldots (81)$ $[where\ x \geq 0]$

Then, equations (80) and (81) =>

$$-x + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - x^2} \geq 0 \ldots (82)$$

$$or, \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - x^2} \geq x$$

$$or, x \leq \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - x^2}$$

$$or, x^2 \leq \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - x^2$$

$$or, 2x^2 \leq \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$$

$$or, x^2 \leq \frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$$

$$\therefore 0 \leq x \leq \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (83)$$

$$\therefore 0 \leq v_{IN1} - v_{IN2} \leq \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \ldots (84)\ [by\ (81)]$$

Case-II: $v_{IN1} - v_{IN2} < 0$

Let, $v_{IN1} - v_{IN2} = -x$ ... (85) $[where\ x > 0]$

Then, equations (80) and (85) =>

$$-(-x) + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - x^2} \geq 0$$

$$or, x + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - x^2} \geq 0 \text{ ... (86)}$$

Inequality (86) holds for

$$0 < x \leq \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \text{ ... (87)}$$

$$or, 0 > -x \geq -\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$$

$$or, 0 > v_{IN1} - v_{IN2} \geq -\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}\ [by\ (85)]$$

$$\therefore\ -\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \leq v_{IN1} - v_{IN2} < 0 \text{ ... (88)}$$

Inequalities (84) and (88) =>

$$-\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \leq v_{IN1} - v_{IN2} \leq \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \text{ ... (89)}$$

Since for the root v_2 given by equation (12*b*) to be admissible, both MOSFET M_1 and MOSFET M_2 must operate together (because otherwise, the square-law relating the current and the voltage of a MOSFET will not hold for the MOSFET which is off),

∴ the domain of admissibility of the root v_2 will be the intersection of the domains (75) and (89).

∴ The root v_2 given by equation (12*b*) will be admissible in the domain

$$-\sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \leq v_{IN1} - v_{IN2} \leq \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \text{ ... (90)}$$

Now, to find out the currents and voltages in the domain $\mathbf{-\sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \leq v_{IN1} - v_{IN2} \leq \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}}$**:**

Here,

$$v = v_2 = \frac{(v_{IN1}+v_{IN2}-2V_{Th})-\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(v_{IN1}-v_{IN2})^2}}{2} \ldots (12b)$$

Equation (2) =>

$$i_{D1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{IN1} - v - V_{Th})^2$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{IN1} - v_2 - V_{Th})^2$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[v_{IN1} - \frac{(v_{IN1}+v_{IN2}-2V_{Th})-\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(v_{IN1}-v_{IN2})^2}}{2} - V_{Th}\right]^2$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[\frac{2v_{IN1}-(v_{IN1}+v_{IN2}-2V_{Th})+\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(v_{IN1}-v_{IN2})^2}-2V_{Th}}{2}\right]^2$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[\frac{2v_{IN1}-v_{IN1}-v_{IN2}+2V_{Th}+\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(v_{IN1}-v_{IN2})^2}-2V_{Th}}{2}\right]^2$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[\frac{(v_{IN1}-v_{IN2})+\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(v_{IN1}-v_{IN2})^2}}{2}\right]^2$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\times\frac{1}{4}\left[(v_{IN1} - v_{IN2}) + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}\right]^2$$

$$= \frac{1}{8}\mu_n C_{ox}\frac{W}{L}\left[(v_{IN1} - v_{IN2}) + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}\right]^2$$

$$= \frac{1}{8}\mu_n C_{ox}\frac{W}{L}\left[(v_{IN1} - v_{IN2})^2 + \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2 + 2(v_{IN1} - v_{IN2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}\right]$$

$$= \frac{1}{8}\mu_n C_{ox}\frac{W}{L}\left[\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} + 2(v_{IN1} - v_{IN2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}\right]$$

$$= \frac{1}{8}\mu_n C_{ox}\frac{W}{L}\times\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} + \frac{1}{8}\mu_n C_{ox}\frac{W}{L}\times 2(v_{IN1} - v_{IN2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}$$

$$or, i_{D1} = \frac{I_{SS}}{2} + \frac{1}{4}\mu_n C_{ox}\frac{W}{L}(v_{IN1} - v_{IN2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2} \ldots (91)$$

Equation (4) =>

$$i_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{IN2} - v - V_{Th})^2$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{IN2} - v_2 - V_{Th})^2$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[v_{IN2} - \frac{(v_{IN1}+v_{IN2}-2V_{Th}) - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2}}{2} - V_{Th}\right]^2$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[\frac{2v_{IN2} - (v_{IN1}+v_{IN2}-2V_{Th}) + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2} - 2V_{Th}}{2}\right]^2$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[\frac{2v_{IN2} - v_{IN1} - v_{IN2} + 2V_{Th} + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2} - 2V_{Th}}{2}\right]^2$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[\frac{(v_{IN2} - v_{IN1}) + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2}}{2}\right]^2$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\times\frac{1}{4}\left[-(v_{IN1} - v_{IN2}) + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}\right]^2$$

$$= \frac{1}{8}\mu_n C_{ox}\frac{W}{L}\left[-(v_{IN1} - v_{IN2}) + \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}\right]^2$$

$$= \frac{1}{8}\mu_n C_{ox}\frac{W}{L}\left[(v_{IN1} - v_{IN2})^2 + \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2 - 2(v_{IN1} - v_{IN2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}\right]$$

$$= \frac{1}{8}\mu_n C_{ox}\frac{W}{L}\left[\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - 2(v_{IN1} - v_{IN2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}\right]$$

$$= \frac{1}{8}\mu_n C_{ox}\frac{W}{L}\times\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - \frac{1}{8}\mu_n C_{ox}\frac{W}{L}\times 2(v_{IN1} - v_{IN2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}$$

$$or, i_{D2} = \frac{I_{SS}}{2} - \frac{1}{4}\mu_n C_{ox}\frac{W}{L}(v_{IN1} - v_{IN2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2} \ldots (92)$$

Equation (6) =>

$$v_X = V_{DD} - i_{D1}R_D$$

$$or, v_X = V_{DD} - \left[\frac{I_{SS}}{2} + \frac{1}{4}\mu_n C_{ox}\frac{W}{L}(v_{IN1} - v_{IN2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}\right] \cdot R_D$$

$$or, v_X = V_{DD} - \frac{I_{SS}}{2} \cdot R_D - \frac{1}{4}\mu_n C_{ox}\frac{W}{L}(v_{IN1} - v_{IN2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2} \cdot R_D \ldots (93)$$

Equation (7) =>

$$v_Y = V_{DD} - i_{D2}R_D$$

$$or, v_Y = V_{DD} - \left[\frac{I_{SS}}{2} - \frac{1}{4}\mu_n C_{ox}\frac{W}{L}(v_{IN1} - v_{IN2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}\right] \cdot R_D$$

$$or, v_Y = V_{DD} - \frac{I_{SS}}{2} \cdot R_D + \frac{1}{4}\mu_n C_{ox}\frac{W}{L}(v_{IN1} - v_{IN2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2} \cdot R_D \ldots (94)$$

(93) – (94) =>

$$v_X - v_Y = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{IN1} - v_{IN2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2} \cdot R_D \ldots (95)$$

Thus, the currents and voltages in various domains are as given in the following table (Table-2):

Table-2: Currents and Voltages in the MOSFET Differential Amplifier Circuit of Fig. 1 in Various Domains

Variable	Domain		
	$(v_{IN1} - v_{IN2}) \geq \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$ **(M_2 off, M_1 on)**	$(v_{IN1} - v_{IN2}) \leq -\sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$ **(M_1 off, M_2 on)**	$-\sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \leq (v_{IN1} - v_{IN2}) \leq \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$ **(M_1 on, M_2 on)**
v	$v_{IN1} - \left(V_{Th} + \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}\right)$	$v_{IN2} - \left(V_{Th} + \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}\right)$	$\frac{(v_{IN1}+v_{IN2}-2V_{Th}) - \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1}-v_{IN2})^2}}{2}$
i_{D1}	I_{SS}	0	$\frac{I_{SS}}{2} + \frac{1}{4}\mu_n C_{ox}\frac{W}{L}(v_{IN1} - v_{IN2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}$
i_{D2}	0	I_{SS}	$\frac{I_{SS}}{2} - \frac{1}{4}\mu_n C_{ox}\frac{W}{L}(v_{IN1} - v_{IN2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}$
v_X	$V_{DD} - I_{SS}R_D$	V_{DD}	$V_{DD} - \frac{I_{SS}}{2}\cdot R_D - \frac{1}{4}\mu_n C_{ox}\frac{W}{L}(v_{IN1} - v_{IN2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}\cdot R_D$
v_Y	V_{DD}	$V_{DD} - I_{SS}R_D$	$V_{DD} - \frac{I_{SS}}{2}\cdot R_D + \frac{1}{4}\mu_n C_{ox}\frac{W}{L}(v_{IN1} - v_{IN2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}\cdot R_D$
$v_X - v_Y$	$-I_{SS}R_D$	$I_{SS}R_D$	$-\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{IN1} - v_{IN2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (v_{IN1} - v_{IN2})^2}\cdot R_D$